SINGLE-PHASE INDUCTION MOTORS

1. Introduction

The single-phase induction motor displays less satisfactory operating characteristics than the polyphase machines, but has achieved, a wide field of usefulness on applications where a polyphase supply is not available. In the home, washing machines, refrigerators, heat-circulating pumps, clocks, kitchen appliances, hand power tools, phonographs, and so on, all are driven by single-phase ac motors.

When a single-phase stator winding is connected to the supply voltage, the resultant field alternates along one space axis only. Such a pulsating field acting on a stationary squirrel-cage rotor cannot produce rotation. Once the rotor turning, however, the rotor produces a cross flux that is in both space and time quadrature with the main-axis flux. These are the two conditions necessary to produce a rotating field. Hence, once the machine is started, the single-phase motor will continue in rotation.

Two Theories may be used to describe the single phase induction motor

i- The cross field theory

ii- The double-revolving field thery.

2. Types of single-phase IM

a. Split-phase

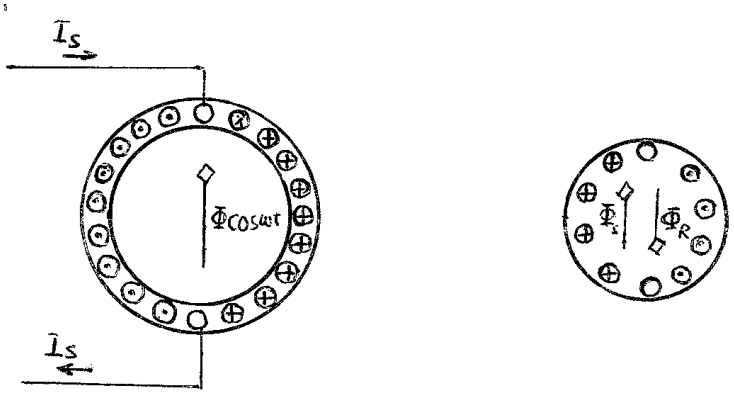
b. Capacitor-Start

c. Shaded-pole

d. Repulsion-Start

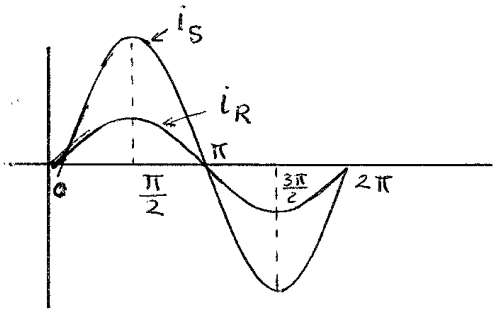
3. Construction

The construction of a two-pole single-phase stator is shown in Fig. 1(a). Windings are distributed much in the fashon of a polyphase motor.

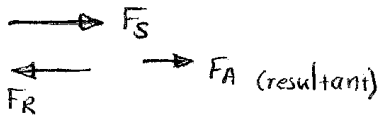


STATOR ROTOR

A squirrel-cage rotor is inserted in the stator. With the rotor at rest and the stator excited, flux will induce a voltages in the rotor, the direction of ~~current~~ flux established by the rotor currents opposes the stator flux. There is no torque-producing angle and the rotor will not revolve, (the motor will be like a static transformer machine).



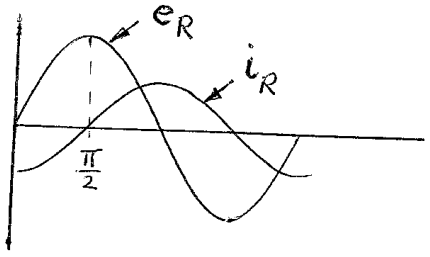
} currents due to alternating flux



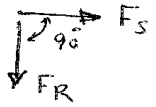
The same machine is now considered but with the rotor revolving:-

(i) A rotational e.m.f (e_R) is generated in the rotor conductors due to the air-gap flux. (same principle as d.c. machine).

(ii) Since in an a.c motors $X_R >> R_R$, the rotational e.m.f e_r will circulate a current in the rotor that is lag e_R by $\pi/2$.



} A.C quantities due to rotation



(iii) The rotor current i_R will produce an m.m.f which is 90° apart from the stator m.m.f., these two m.m.f are sufficient to establish a rotating m.m.f, and therefore magnetic field is established in a single-phase induction motor when its rotor is revolving.

N.B:

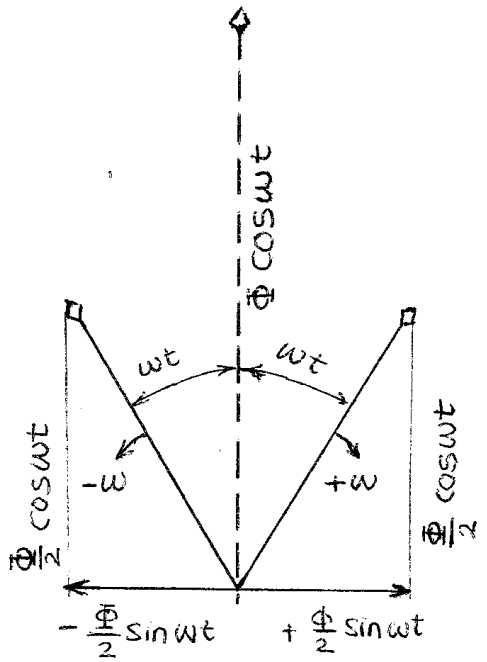
1) The rotating field is of variable magnitude through out each revolution

2) The rotating field can be obtained in either direction depending on the original turning effort given to the rotor.

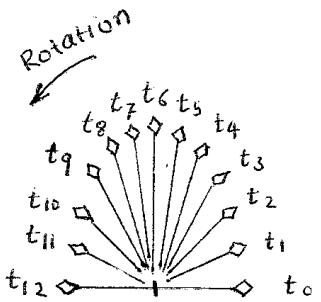
Double-Revolving Field Theory.
Equivalent Circuit of 1-phase IM

1. The double-revolving field Theory.

The double revolving flux theory permits application of principles of the poly phase motor to the single-phase machine. In Fig. 1(a), two fluxes are shown, each with the magnitude of $\Phi/2$ and rotating at angular velocity ω but in opposite directions.



(a) Double-revolving Field



Revolving flux

Considering only the flux $\Phi/2$ rotating at $+\omega$, this flux would produce T-S ch/s curve as shown in Fig.1b [$\Phi/2(+\omega)$]. Similarly, when $\Phi/2$ rotating at $-\omega$ is seperately considered, the T-S curve is the same except that both torque and speed are negative. [$\Phi/2(-\omega)$ curve].

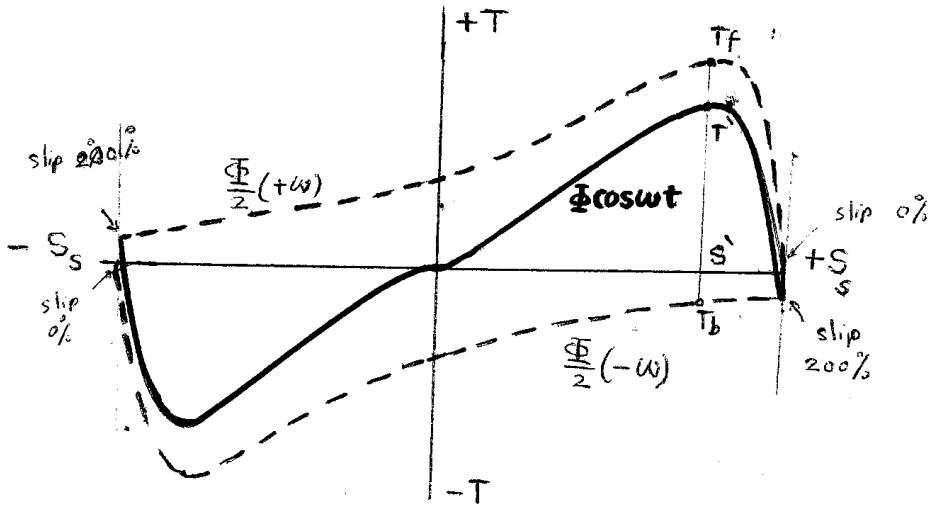


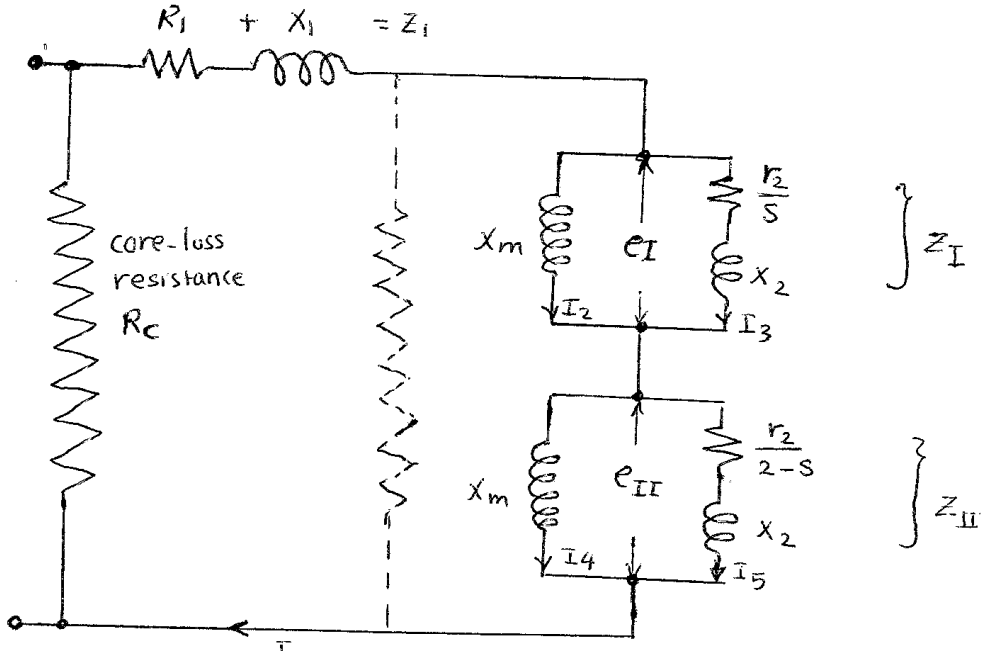
Fig. 1 (b) Torque - Speed characteristics

The fluxes cannot be isolated - both are present at all times, and the sum of the curves is representative of the combined characteristics. This sum is shown by the solid curve [$\Phi\cos\omega t$]. The above explanation holds equally well for operation in either direction, since the motor runs in the direction of the demand of starting system.

The equations and diagrams are built up by assuming that the actual stator and rotor of the single-phase motor are replaced by two stators and rotors, with the stator in series and the rotors on the same shaft. It is assumed that one rotor turns with its magnetic field and the other runs against its magnetic field.

2. Equivalent Circuit.

According to the Double-Revolving field theory, the equivalent network of 1φ IM is made up of a stator primery winding and of two imaginary rotors.



R_1 , X_1 are Stator resistance and reactance respectively

r_2 , X_2 are rotor resistance and reactance respectively. (half the actual value)

x_m half magnetising reactance of the rotor.

s , $2-s$ are slip for the two fictitios rotors.

The forward torque is

$$T_f = I_3^2 \frac{r_2}{s} \quad \text{(synchronous watts)}$$

The backword torque is

$$T_b = I_5^2 \frac{r_2}{2-s} \quad \text{(synchronous watts)}$$

The resultant torque :

$$T = T_f - T_b$$

Example

A single-phase $\frac{1}{4}$ HP, 4-pole motor operating on 60-cycles (Hz), with 110 volts supply, having :

$R_1 = 1.86\ \Omega$ $X_1 = 2.56\ \Omega$

Friction and windage losses = 13.5 Watts

$R_2 = 3.56\ \Omega$ $X_2 = 2.56\ \Omega$

$X_m = 53.5\ \Omega$

For a slip of 0.05 calculate the performance

Of the motor.

Solution

Impeadance of forward rotor :-

$$Z_I = \frac{j\frac{X_m}{2}\left(\frac{R_2}{2S} + j\frac{X_2}{2}\right)}{\frac{R_2}{2S} + j\left(\frac{X_2 + X_m}{2}\right)}$$

$$= \frac{j\,26.7\left(\frac{1.78}{0.05} + j\,2.56\right)}{\frac{1.78}{0.05} + j\,(2.56 + 26.7)}$$

$$= 12.4 + j\,17.15 \quad \text{or} \quad 21.15\ \Omega .$$

Impeadance of the backword rotor :-

$$Z_{II} = \frac{j\frac{X_m}{2}\left(\frac{R_2}{2-S} + jX_2\right)}{\frac{R_2/2}{(2-S)} + j\frac{1}{2}(X_2 + X_m)}$$

$$= 0.84 + j\,1.26 \quad \text{or} \quad 1.51\ \Omega .$$

the total impeadance $Z_T = Z_1 + Z_I + Z_{II}$

$\therefore Z_T = 15.10 + j\,20.97 \quad \text{or} \quad 25.85\ \Omega .$

$$I_1 = \frac{110}{25.85} \quad \text{or} \quad 4.27 \text{ amp.}$$

The power factor :

$$P.f = \frac{R_{total}}{Z_T} = \frac{15.10}{25.85} \quad \text{or} \quad 0.585$$

To find the branch currents :

$$e_I = I_1 Z_I = 4.27 \times 21.15 = 90.4 \text{ V.}$$

$$e_{II} = I_1 Z_{II} = 4.27 \times 1.51 = 6.44 \text{ V.}$$

$$Z_3 = \sqrt{\left(\frac{R_2}{5}\right)^2 + \left(\frac{X_2}{2}\right)^2} \quad \text{or} \quad 35.7 \ \Omega$$

$$\therefore \quad I_3 = \frac{e_I}{Z_3} = \frac{90.4}{35.7} = 2.53 \text{ amp.}$$

$$Z_5 = \sqrt{\left(\frac{R_2/2}{2-5}\right)^2 + \left(\frac{X_2}{2}\right)^2} \quad \text{or} \quad 1.57 \ \Omega$$

$$\therefore \quad I_5 = \frac{e_{II}}{Z_5} = \frac{6.44}{1.57} = 4.1 \text{ amp.}$$

$$T_f = I_3^2 \cdot \frac{R_2/2}{5} = 228 \text{ synch. watts.}$$

$$T_b = I_5^2 \frac{R_2/2}{2-5} = 15.3 \text{ synch. watt.}$$

$$T = T_f - T_b = 212.7 \text{ synch. watt.}$$

Output in watts = synch. watts $\times (1-5)$

$$\therefore \quad = 212.7 \times 0.95 = \underline{202} \text{ watts}$$

Net output = Output in watts $-$ losses $= 202 - 13.5 = 188.5$ watts

$$\% \text{ efficiency} = \frac{\text{output}}{\text{input}} = \frac{188.5}{110 \times 4.27 \times 0.585} \times 100 = \underline{68\%}.$$

TYPES OF SINGLE-PHASE INDUCTION MOTORS

1. SPLIT-PHASE MOTOR

The stator has two windings, a main and auxiliary. The latter is energized during the starting period only.

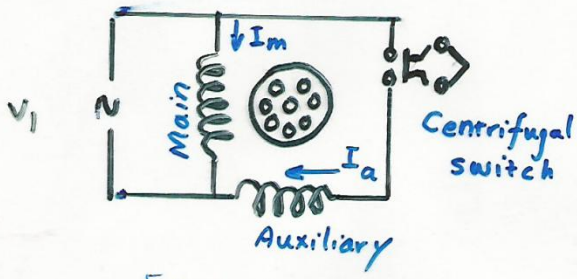


Fig. 1
Schematic diagram

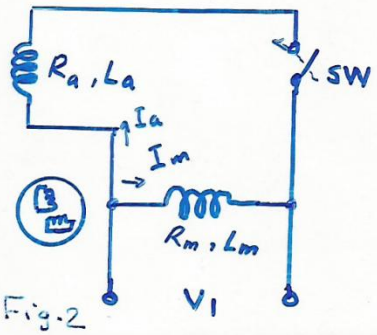


Fig. 2
circuit model

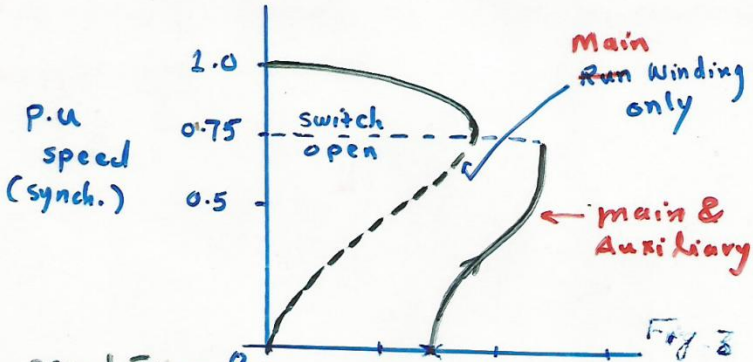


Fig. 3

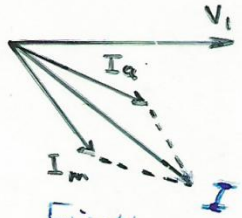
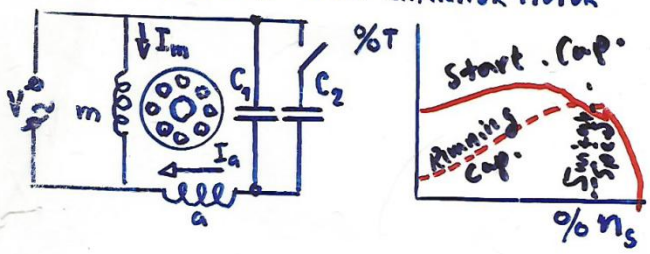


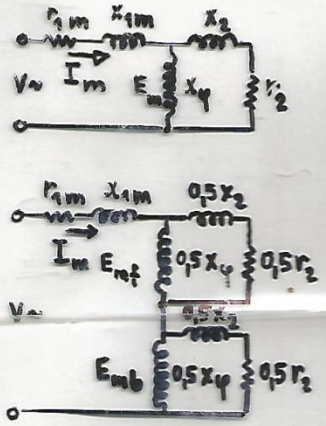
Fig. 4
phasor diagram at starting

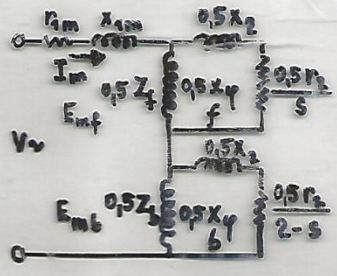
When the rotor has accelarated to about 75 to 80% of its synchronous speed, a centrifugal switch opens to disconnect the auxiliary winding.

THE PERMANENT-SPLIT-CAPACITOR MOTOR



EQUIVALENT CIRCUIT





$Z_f = R_f + jX_f \equiv \left(\frac{r_2}{s} + jx_2\right)$ IN PARALLEL WITH jx_φ

$Z_b = R_b + jX_b \equiv \left(\frac{r_2}{2-s} + jx_2\right)$ —"—

$$R_f = \frac{x_\varphi^2}{x_{22}} \frac{1}{sQ_2 + (1/sQ_2)}$$

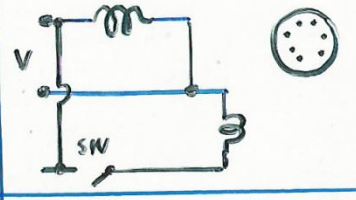
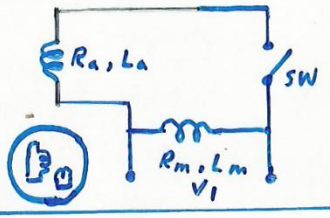
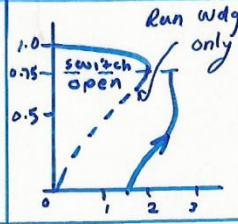
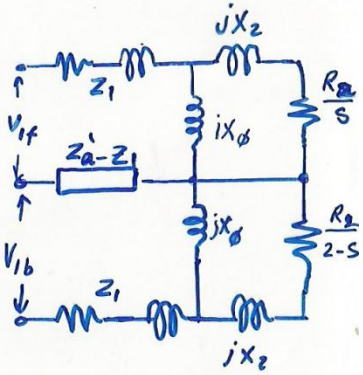
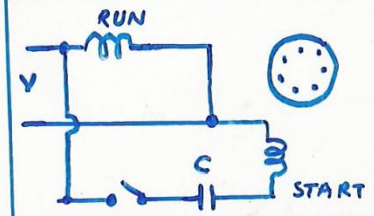
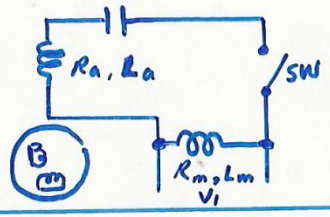
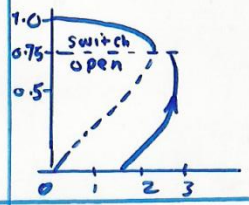
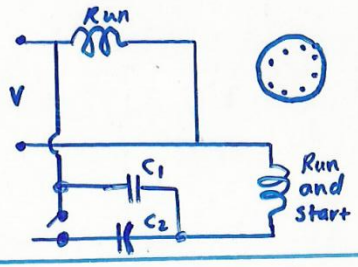
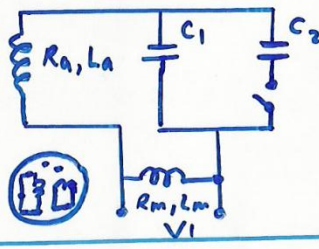
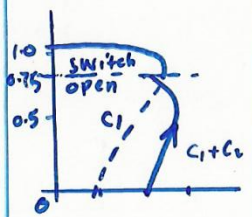
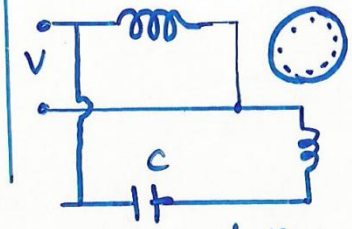
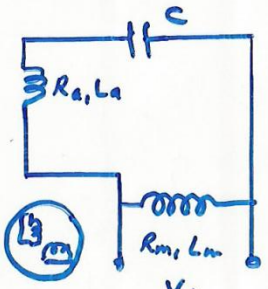
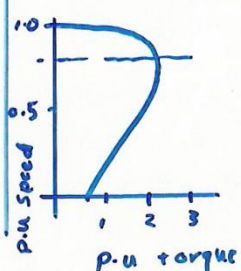
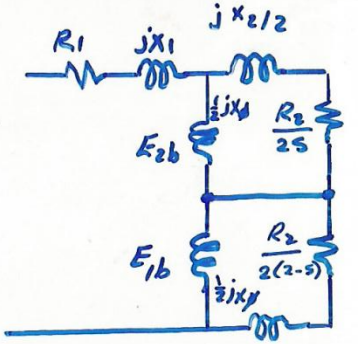
$$X_f = \frac{x_2 x_\varphi}{x_{22}} + \frac{R_f}{sQ_2}$$

$$x_{22} = x_2 + x_\varphi \qquad Q_2 = \frac{x_{22}}{r_2}$$

$$R_b = \frac{r_2}{2-s}\left(\frac{x_\varphi}{x_{22}}\right)^2$$

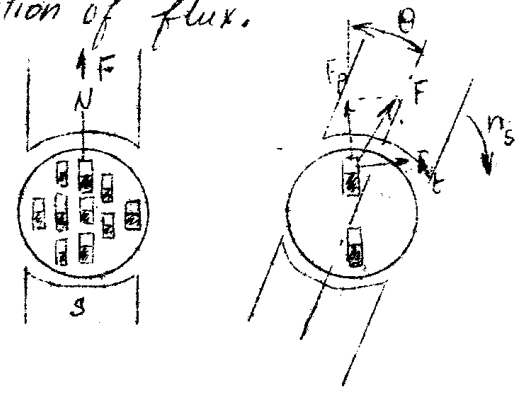
$$X_b = \frac{x_2 x_\varphi}{x_{22}} - \frac{R_b}{(2-s)Q_2}$$

SINGLE PHASE IM

Type	Schematic	Circuit Model	Speed-Torque chg	Equivalent CCT
SPLIT PHASE				 Equivalent CCT during Starting, (Also applied for capacitor run motor during running).
APACTOR TART				
APACITUR TART & UN anent-Split apacitor)				
ERMANENT APACITOR	 Start and Run connection			 Eq. CCT After Starting. Applic for all 1φ motor during running.

HYSTERESIS MOTOR

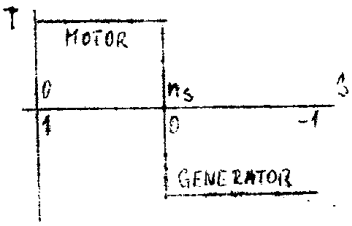
The phenomenon of hysteresis can be used to produce mechanical torque. In this simplest form, the rotor of a hysteresis motor is a smooth cylinder of magnetically hard steel, without rotor winding or teeth. It is placed within a slotted stator carrying distributed windings designed to produce as possible a sinusoidal space distribution of flux.



The rotor can be imagined as a set of elementary magnets and the revolving magnetic field as permanent magnets revolving at synchronous speed n_s. Elementary magnets and revolving field are coupled by magnetic force F acting in the sense of magnetic axis.

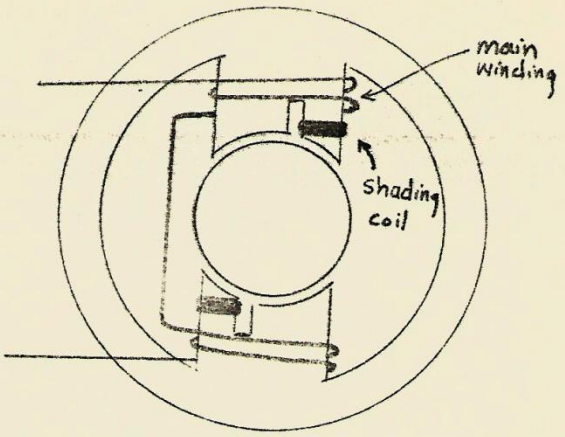
When the main field is revolving, the elementary magnets will be turning too. But due to the hysteresis delay (molecular friction), the turning does not occure immediately but appears some angle Θ. In this case the force F can be resolved into components F_p, F_t. The radial component F_t creates so called „HYSTERESIS TORQUE". When the angle Θ is increasing the radial component is increased too. $F_p = F \cos\Theta$, $F_t = F \sin\Theta$.

The angle Θ is given by rotor material, and the value of hysteresis torque is given by coercitive force and does not depend on the speed. The torque speed characteristic is on figure. When an external torque is applied and the speed is $n > n_s$ than the motor operates as generator. Beside the hysteresis torque an eddy currents torque is created. This torque has its maximum value for $s = 1$ and at $s = 0$ is zero.

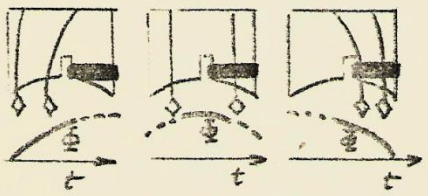


The hysteresis motor is inherently quiet and produce smooth rotation. The motor can be made as multispeed operated by pole changing connection.

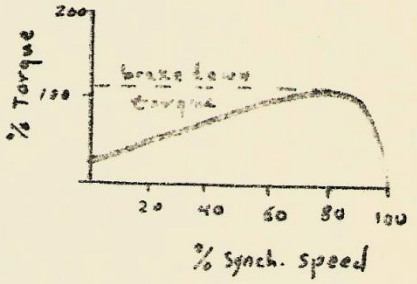
SHADED POLE MOTOR



(a) construction



(b) Flux movment



(c) Typical Torque/speed chls.

The shaded-pole motor has a low p.f, is inefficient (η = 30 - 40%), and has low breakdown torque (about 125% rated torque). However, it is cheapest motor to produce. In the range from $\frac{1}{25}$ to $\frac{1}{4}$ H.P, where efficiency is a minor concern, it is employed in devices such as toys, fans, and phonographs turn-tables.

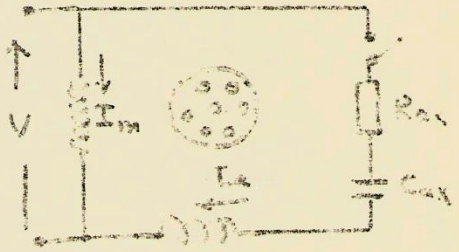
Problem:

A single-phase capacitor induction motor has two identical winding having impedances:

$$Z_m = 20 + j\,90 \quad \Omega$$
$$Z_a = 20 + j\,90 \quad \Omega$$

Windings are supplied from a.c. source of 220 V, 50 Hz. Calculat C_{ax} and R_{ax} necessary to introduce in the auxiliary winding to obtain a revolving magnetic field.

Solution:-



In order to obtain revolving magnetic field the currents in the main and auxiliary windings must be same magnitudes but shifted by j in space

hence: $\quad j\dot{I}_m = I_a$

$$j\,\frac{V}{R_m + jX_m} = \frac{V}{(R_a + R_{ax}) + j(X_a - X_{ax})}$$

$$\therefore\ j(R_a + R_{ax}) - (X_a - X_{ax}) = R_m + jX_m$$

Equating Real & Imaginary parts:

1. $R_a + R_{ax} = X_m$ $\qquad \therefore\ R_{ax} = X_m - R_a = 90 - 20 = 70\ \Omega$

2. $R_m = -X_a + X_{ax}$ $\qquad \therefore\ X_{ax} = R_m + X_a = 20 + 90 = 110\ \Omega$

$$\frac{1}{\omega C_{ax}} = 110 \qquad \therefore\ C_{ax} = \frac{1}{110 \times 2\pi \times 50} = 28.9\ \mu F.$$

A 1/3 HP 120 V, 60 Hz capacitor started single-phase a.c. motor has the following constants:-

Main wdg $Z_m = 4.5 + j3.7 \ \Omega$

Aux. wdg $Z_a = 9.5 + j3.5 \ \Omega$

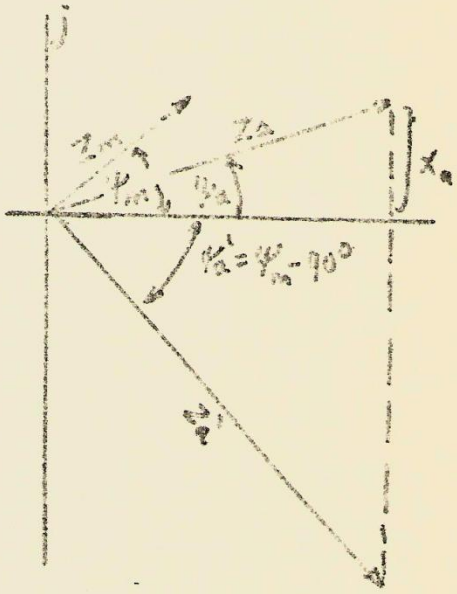
Find the value of the starting capacitor that will placed the main and auxiliary wdg currents in quadrature at starting.

Solution

$\psi_m = \tan^{-1} \frac{3.7}{4.7} = 39.6^\circ$

The impedance angle of the auxiliary wdg must be

$\psi_a' = 39.6 - 90 = -50.4^\circ$



The reactance X_c of the capacitor must satisfy the relationship

$\tan^{-1} \frac{3.5 - X_c}{9.5} = 50.4^\circ$

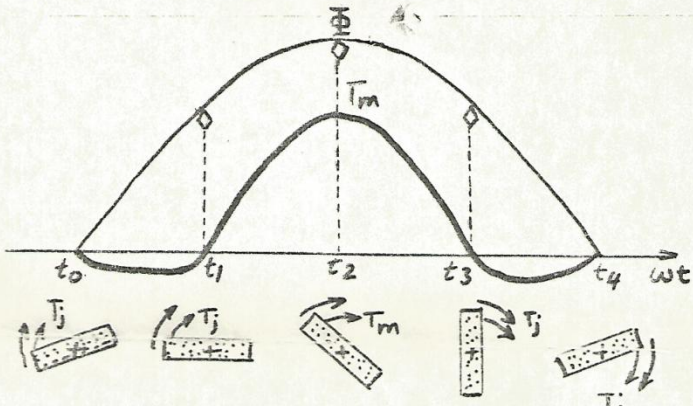
$\frac{3.5 - X_c}{9.5} = -1.21$

$\therefore X_c = 1.21 \times 9.5 + 3.5 = 15 \ \Omega$

$\therefore C = \frac{10^6}{15 \times 2\pi \times 60} = 177 \ \mu F$

RELUCTANCE MOTOR (synchronous)

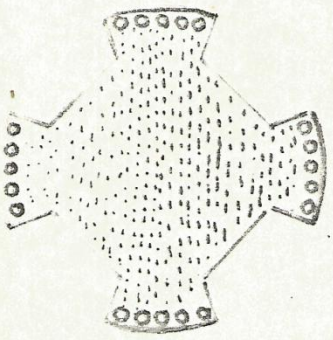
The reluctance principle exploits the characteristic that magnetic flux will select the lowest - reluctance path in completing its circuit



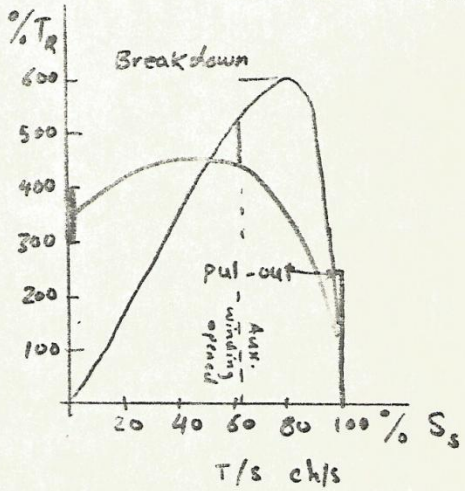
Tm = magnetic torque

Tj = inertia torque

RELUCTANCE PRINCIPLE



4-pole squirrel-cage rotor



Reluctance Motor

A reluctance motor is not self-starting, but it has self-sustaining at synchronous speed. Continuous rotation results from alternating inertial and electromagnetic torques.

EXAMPLE.

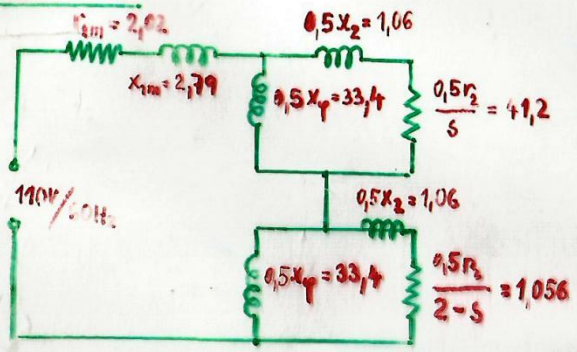
A ¼ HP, 110V, 60Hz, 4 pole single phase induction motor capacitor start, has the following constants and losses:

$r_{1m} = 2{,}02$ ohms $r_2 = 4{,}12$ ohms $x_{eq} = 66{,}8$ ohms Friction and windage losses = 13 watts

$x_{1m} = 2{,}79$ ohms $x_2 = 2{,}12$ ohms Core loss = 24 watts

For $s = 0{,}05$ determine: stator current, power factor, power output, speed, torque and efficiency when this motor is running as a single phase motor at rated voltage and frequency with its starting open.

SOLUTION:



$x_{22} = x_2 + x_\varphi = 68{,}92$ ohms

$Q_2 = \frac{x_{22}}{r_2} = 16{,}728$

$sQ_2 = 0{,}836$

$R_f = \frac{x_\varphi^2}{x_{22}} \cdot \frac{1}{sQ_2 + (1/sQ_2)} = 31{,}9$ ohms

$x_f = \frac{x_1 x_\varphi}{x_{22}} + \frac{R_f}{sQ_2} = 40{,}3$ ohms

$Z_f = (31{,}9 + j\,40{,}3)$ ohms

$R_b = \frac{r_2}{2-s}\left(\frac{x_\varphi}{x_{22}}\right)^2 = 1{,}98$ ohms $X_b = \frac{x_2 x_\varphi}{x_{22}} + \frac{R_b}{(2-s)\,Q_2} = 2{,}12$ ohms $Z_b = (1{,}98 + j\,2{,}12)$ ohm.

$r_{1m} + jX_{1m} = 2{,}02 + j\,2{,}79$

$0{,}5(R_f + jX_f) = 15{,}95 + j\,20{,}15$

$0{,}5(R_b + jX_b) = 0{,}99 + j\,1{,}06$

$Z = 18{,}96 + j\,24 = 30{,}6 \angle 51{,}7°$ Input impedance

Stator current $I_m = \frac{110}{30{,}6} = 3{,}59$ amp. $\left(I_m = \frac{V}{|Z|}\right)$

Power factor $\cos\varphi = \cos 51{,}7° = 0{,}62$; Power input $P_i = UI_m\cos\varphi = 244$ watts

Power absorbed by forward field $P_{gf} = I_m^2 \cdot 0{,}5R_f = 3{,}59^2 \cdot 15{,}95 = 206$ watts

Power absorbed by backward field $P_{gb} = I_m^2\, 0{,}5R_b = 3{,}59^2 \cdot 0{,}99 = 12{,}8$ watts

Internal mechanical power $P = (1-s)\omega_s T = (1-s)(P_{gf} - P_{gb}) = 0{,}95\,(206-13) = 184$ watts

Rotational loss = core loss + friction & windage loss $\Delta P = 24 + 13 = 37$ watts

Power output $P_o = P - \Delta P = 184 - 37 = 147$ watts (0,197 HP)

Synchronous speed $n_s = 1800$ rpm (30 1/sec) ; $\omega_s = 2\pi n_s = 188{,}5$ rad/sec

Rotor speed $n = (1-s)n_s = 0{,}95 \cdot 1800 = 1{,}710$ rpm ($\omega = 0{,}95 \cdot 188{,}5 = 179$ rad/sec)

Torque $T = \frac{P_o}{\omega} = \frac{147}{179} = 0{,}821$ Nm

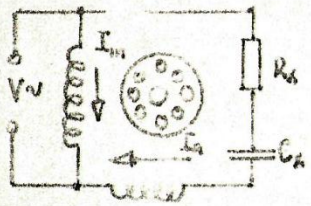
Efficiency $\eta = \frac{P_o}{P_i} = \frac{147}{244} = 0{,}602$

PROBLEM:

A single-phase capacitor induction motor has two identical windings having impedances:

$Z_m = (20 + j\,90)$ ohms Windings are supplied from a.c. source 220V/50c/s

$Z_a = (20 + j\,90)$ ohms Calculate C_x and R_x necessary to introduce in auxiliary winding to obtain circular revolving magnetic field.



The conditions for rotating circular magnetic field is to have currents in main and auxiliary way the same and shifted by 90° in space.

$$\dot{I}_m = I_a$$

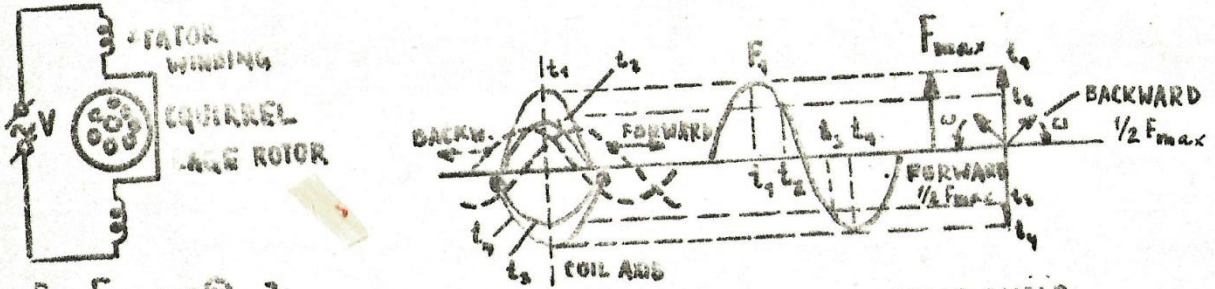
$$\dot{I}\frac{V}{R_m + jX_m} = \frac{V}{(R_a + R_x) + j(X_a - X_c)} \quad ; \quad (R_a + R_x) - (X_a - X_c) = R_m + jX_m$$

real and imaginary parts must be equal

1. $R_a + R_x = X_m \qquad R_x = X_m - R_m = 90 - 20 = 70$ ohms

2. $R_m = -X_a + X_c \qquad X_c = R_m + X_a = 20 + 90 = 110$ ohms

$$\frac{1}{\omega C_x} = 110 \Rightarrow C_x = \frac{1}{110 \cdot 314} = 2{,}84 \cdot 10^{-5} \qquad C_x = 28{,}4\ \mu F$$



$F_1 = F_{max} \cos\Theta$

$F_{max} = F \cos\omega t$

$F_1 = F \cos\omega t \cos\Theta$

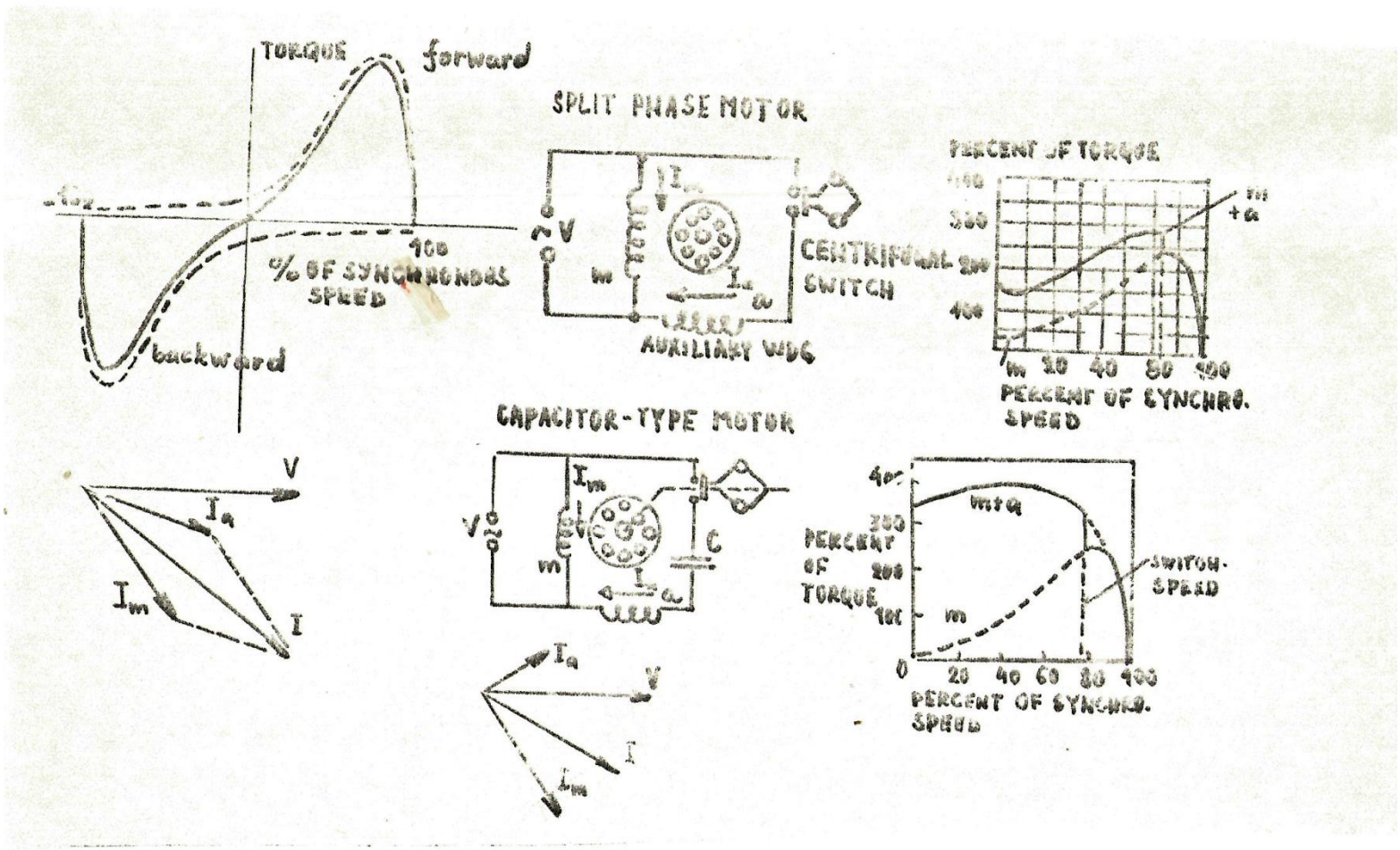
Θ... ELECTRICAL SPACE ANGLE

F_{max} ... INSTANTANEOUS VALUE OF MMF

$$F_1 = \frac{1}{2} F \cos(\Theta - \omega t) + \frac{1}{2} F \cos(\Theta + \omega t)$$

FORWARD BACKWARD

F... PEAK VALUE CORRESP. TO MAXIMUM INSTANTENEOUS CURRENT


TORQUE
forward
backward
100
% OF SYNCHRONOUS SPEED
SPLIT PHASE MOTOR
V
m
a
CENTRIFUGAL SWITCH
AUXILIARY WDG
PERCENT OF TORQUE
m+a
m
PERCENT OF SYNCHRO. SPEED
CAPACITOR-TYPE MOTOR
C
V
I_a
I_m
I
PERCENT OF TORQUE
m+a
m
SWITCH-SPEED
0
20
40
60
80
100
PERCENT OF SYNCHRO. SPEED

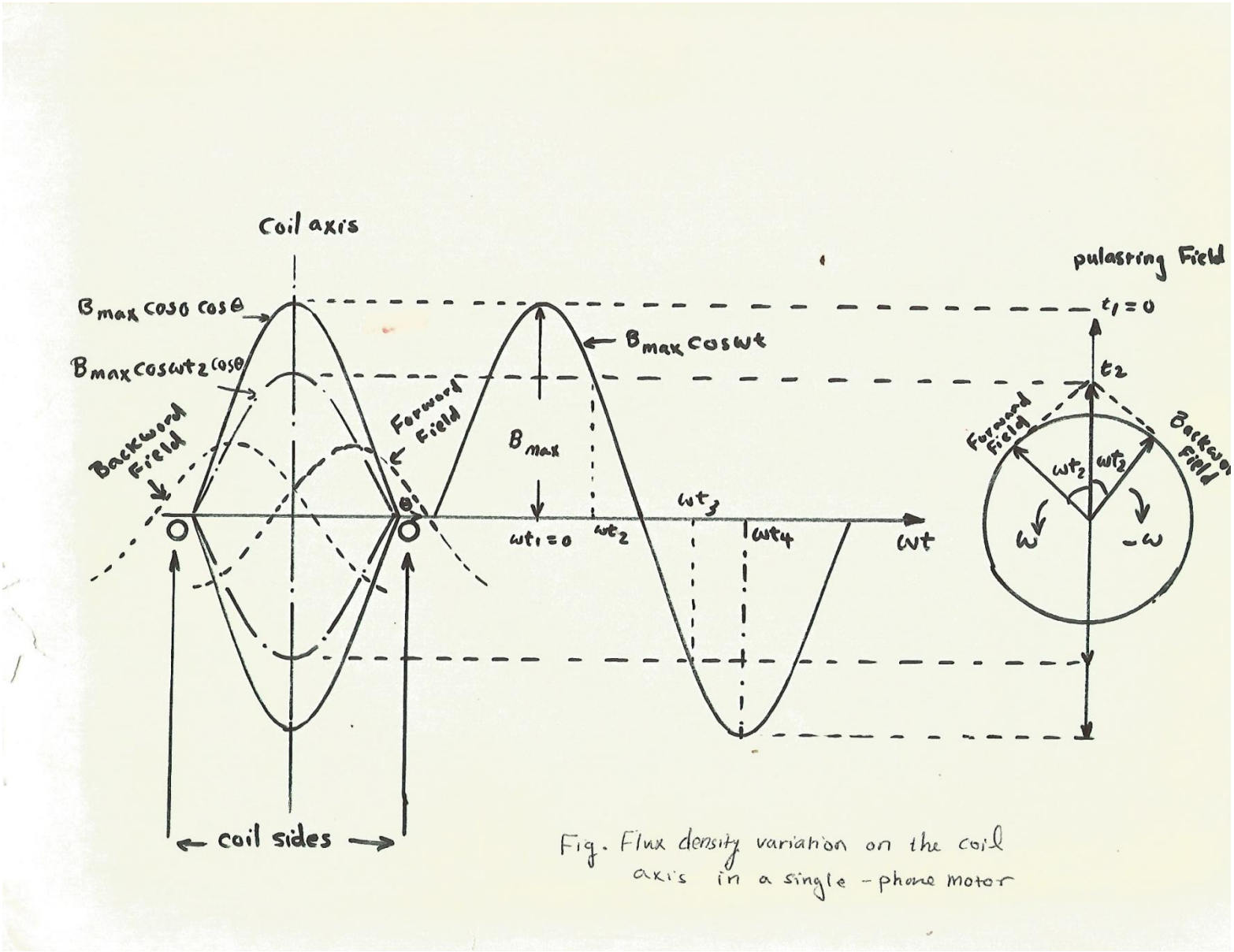


Fig. Flux density variation on the coil axis in a single-phase motor